Magnetic Torque and Force Experiments
Physics 392

This lab will allow you to examine two very important properties of magnetic dipoles:

(i) A magnetic dipole feels a torque, \( \vec{\tau} = \vec{\mu} \times \vec{B} \), where \( \mu \) is the dipole moment and \( B \) is the applied uniform magnetic field.

(ii) A magnetic dipole does not feel any net force in a uniform magnetic field. In a non-uniform magnetic field the force on the dipole, in the \( z \) direction, is given by

\[
F_z = \mu \frac{dB_z}{dz}.
\]

The Experiments
You will do a total of four experiments. In all cases the magnetic dipole being used is a small permanent magnet. In the first three experiments this magnet is embedded in plastic cue ball.

The magnetic field in the experiments is supplied by "Helmholz Coils". (Helmholz Coils are a set of coils configured in a way that maximizes the uniformity of the B field they generate. You will discover their properties more completely once the experiments get underway.)

In all the experiments the main goal is to obtain and plot data for a number of different B fields, or B gradients and see if the functional dependences predicted by the above equations are observed experimentally.

A Static Torque. You will test the validity of the torque relation by balancing magnetic torque with gravitational torque.

B Harmonic Oscillation. You will test the validity of the torque relation by displacing a dipole from its equilibrium position and seeing if the frequencies of oscillation are in agreement with theory.

C Precessional Motion. You will compare experimental observations of a spinning magnetic dipole in a magnetic field with what the torque relation predicts.

D Force in a Magnetic Field Gradient. You will test the validity of the force relation by placing a dipole in a known field gradient and measuring the force exerted on it.

Before attempting any of the experiments be sure to complete the pre-lab assignments given below! Staple these assignments into your lab notebooks.
MAGNETIC FORCE*

MF1-A

STUDENT EXPERIMENTS

A PRODUCT OF TEACHSPIN, INC.

Written by Jonathan F. Reichert

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OVERVIEW

“Magnetic Force”, MF1-A is a hands-on instrument designed to explore the forces on a permanent magnetic dipole moment $\mu$ caused by various external magnetic fields. Essentially all of your introductory physics texts discuss magnetic forces on a magnetic dipole, yet you rarely (if ever) have an opportunity to perform either qualitative or quantitative experiments of these interactions. TeachSpin, Inc. designed “Magnetic Force” to give you a first-hand learning experience with magnetic forces. We are convinced that the Magnetic Force experiments will surprise (sometimes even amaze) most of you. But we are also certain that these experiments will serve to clear up one of the most pervasive misconceptions in all of electromagnetism.

THE INSTRUMENT

“Magnetic Force” MF1-A consists of a pair of coils mounted in the so called “Helmholtz” configuration and a small neodymium-iron-boron disk permanently magnetized along its cylindrical axis. This disk is the model system for an ideal magnetic dipole moment. It is mounted in a Delrin plastic gimbals so it is free to rotate about the gimbals axis, and is suspended by a long precision nonmagnetic spring, as shown in the diagram. The opposite end of the spring connects to a brass rod which is held in place on the top of the transparent plastic tower by a thumb screw in the Delrin cap. The spring conforms to Hook’s law within the experimental error of the apparatus. However, you must calibrated it for your measurements, since the spring constant may change over time due to mishandling and permanent elastic deformation.
The instrument requires an external variable power supply capable of passing 3 amperes through a resistance of 5.6 ohm, or about 17 volts. You should not exceed 3 amperes for a significant time since the coils will overheat and could cause damage to you and the instrument. Current regulated power supplies are a distinct advantage for these experiments, since the resistance of the copper wires significantly increases with temperature. If the supply is only voltage regulated, this voltage will need to be adjusted throughout the experiment to maintain the same current, as the coils warm due to the ohmic heating. Remember, the magnetic field only depends on the current, for a given set of windings. You will also need a centimeter scale or calipers, one capable of measuring to 0.5 mm accuracy. The ball bearings have a mass close to 1g, but slightly better accuracy in calibrating the spring constant can be obtained by weighing all five of them to an accuracy of ± .1g.

MF1-A SPECIFICATIONS:

Two coils
168 turns/coil, #20 formvar insulated copper wire
Mean radius 7.0 cm
Coil separation 7.0 cm (Helmholz conditions)
Maximum current 3.0A (for longtime periods)
Coil Resistance 2.8 Ω/coil
Coils are surge protected against back emf
Positive current flowing into the red binding post will produce a magnetic field in the upward vertical direction along the coils axis’s. This is true for both the upper and lower coils.
Spring constant ~ 1N/m
Magnetic Moment ~ 4 A-m²

Note: Do not operate the coils in parallel. The current will not necessarily divide equally and thus give rise to unpredictable field.
LAB PREPARATION-BEFORE LAB BEGINS

Before coming to the lab you should do some theoretical calculations of the magnetic fields that can be created by one or two coils of wire which have a steady current passing through them. Go to your textbook and refresh your understanding of the magnetic field along the axis of a single loop of wire of radius R with a circulating current of I amperes.

What is the expression of the magnetic field B at the at the point z along the axis of the loop? What are the units of R, I, z, and B?

Supposing there are N turns of wire, all in close proximity to the first turn, all carrying the same current I. What is the expression for the magnetic field for these N turns?

Suppose we consider the magnetic field generated by two identical coils of N turns, radius R, and with the same current I passing through both coils. Suppose we examine this field in the central region when the coils are separated by a distance equal to their radius.

a) First, find the expression for the axial magnetic field as a function of current at the exact center of the coil set. Which direction does it point?

b) Calculate the numerical value of the magnetic field for the maximum current of 3 amperes.

c) Calculate the numerical values of the magnetic field for six additional points at ±1 cm, ±2 cm, ±3 cm about the center, along the z-axis for a current of 3 amperes.
d) Plot the magnetic field as a function of distance along the z-axis using the data you calculated above. What can you infer from this plot?

Now you should consider the same coils but connected in such a way that the current flows in opposite directions in the two coils. Repeat all the calculation in parts a, b, c, d for this configuration of current as well as the following additional calculations:

e) Calculate the rate of change of the magnetic field with distance $\frac{\Delta B_z}{\Delta z}$ as a function of $I$, from the slope of the curve plotted in part (d).

f) Calculate $\frac{dB_z}{dz}$ for a single loop of wire by differentiating your expression for the axial magnetic field of a single loop of current.

g) Calculate the expression for $\frac{dB_z}{dz}$ two coils each with $N$ turns and radius $R$ separated by distance $R$.

h) Compare the calculation of $\frac{dB_z}{dz}$ in part g, with the values of $\frac{\Delta B_z}{\Delta z}$ determined in part c.

All these calculations, as well as a review of the discussion of the interaction of a magnetic moment $\mu$ with a magnetic field in your textbook, are necessary before beginning the actual experiments. You should understand what caused a torque and what causes a force on a magnetic dipole. You should know the mathematical expressions for both. Of course you must keep in mind the distinctions between torque and force and how they are manifested in a real physical system.

EXPERIMENTS

I. Calibrate the spring constant $k$. Use the five 1g steel ball bearings as weights. Be careful of the spring, it can be permanently damaged by over stretching it while you are handling it. Hint: Determine this constant from the slope of a graph. Why use a graph? Explain.

II. Connect the two coil system in series in such a way that the current flows in the same direction in each coil. Connect the coils to a d.c. power supply capable of producing at least 3A at 17V. Adjust the brass rod in such a way that the
magnetic dipole hangs in the central region of the two coil, near zero on the scale. Turn on full current (3 A). What happened? Reverse the current through both coil. What happened? Explain all you observed.

III. Connect the two coil system in series in such a way that the current flows in the opposite direction in each coil. Again, place the magnetic dipole at the center and turn on the supply to 3A through the coils. What do you observe? Reverse the leads to the supply, what do you observe? Explain your observations.

With 3A flowing through the coils, lower the dipole to the bottom of the tower and slowly raise it up (by raising up the brass rod) through the central region to above the top coil. Now lower the dipole down to the bottom of the tower. Repeat this procedure several times, carefully watching and recording qualitatively what happens. Give a brief but clear explanation of what you observed. Hint: this “simple” experiment demonstrates both magnetic torque and magnetic force.

IV. Connect your power supply to the top coil only. Adjust the supply voltage for the maximum current of 3A through the coil. Your job is to measure the magnetic field gradient as a function of axial distance z above the coil, for this loop of wire. Compare your measurement with a theoretical value for this gradient you calculated in the lab preparation part (f). Hint: Part (f) was calculated for a single turn, you must do it for 168 turns.
MAGNETIC TORQUE*
M\(\tau\)1-A
STUDENT LAB MANUAL

A PRODUCT OF TEACHSPIN, INC.

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**Static Torque Experiment** --- Pre-Lab Assignment

OBJECTIVE:
A magnetic dipole experiences a net torque in a uniform magnetic field. Take advantage of this physical principle to determine the magnetic moment of the dipole which is inside the cue ball given to you by your instructor.

QUESTIONS:

1. The picture below shows the Magnetic Torque cue ball floating on the air bearing in an applied uniform magnetic field, $B$. This corresponds to the condition when the currents in the top and bottom coils of the Magnetic Torque instrument are flowing in the same direction. In what direction are the currents flowing if the magnetic field is pointing upward?

![Figure 1](image-url)

2. The magnetic moment, $\mu$, is shown as a vector, as is the magnetic field. Draw on Figure 1 any magnetic torque experienced by the ball, and give the magnitude of this torque in algebraic form.

3. The picture on the next page again shows the ball floating on the air bearing in an applied uniform magnetic field. This time, however, an aluminum rod of mass $M$ carrying a moveable plastic cylindrical weight of mass $m$ has been inserted into the ball. Identify all of the torques experienced by the ball, and give the magnitude of those torques in algebraic form. Do all of the torques act in the same direction? How do we add or subtract torques? (When you're in the lab, you will observe that in the absence of a magnetic field, and without the extra rod, the cue ball is nearly rotationally balanced. Neither the handle nor the embedded magnet produce a significant gravitational torque.)
4. If the magnitude of the magnetic torque is greater than the gravitational torque in the situation shown in Figure 2, will there be a net torque on the ball? If so, will this torque cause the ball to begin to rotate in a particular direction?

5. If the magnitude of the magnetic torque is less than that of the gravitational torque, will there be a net torque on the ball? If so, will this torque cause the ball to begin to rotate in a particular direction?

6. If the magnitude of the magnetic torque is equal to the magnitude of the gravitational torque, what will be the net torque experienced by the ball? Will the ball rotate? Write down this physical situation in algebraic form.

7. In the equation in Question #5, what letters represent known quantities, or quantities that we can directly measure using the Magnetic Torque instrument and standard laboratory equipment? (Hint: The magnitude of the magnetic field is directly proportional to the current, with the proportionality factor being 0.00137 teslas/ampere.)
8. In the equation in Question #5, what is the remaining symbol representing the unknown constant quantity that is not directly measurable?

9. There are two possible methods by which we could calculate this unknown constant quantity. The first is by making a “single-point” measurement. While the ball is in static equilibrium, we could make one measurement of each of the directly measurable quantities involved in our equation, and from those single measurements calculate the unknown constant quantity. But notice that there are two directly measurable quantities that the experimenter can vary. What are these two variables?

10. Since there are two variables, we can make multiple measurements, and from these multiple measurements calculate the unknown constant quantity. This is what we will call a “multiple-point” measurement. If we were to change one of the experimenter-controlled variables, how would we have to adjust the magnitude of the second variable in order to maintain rotational static equilibrium?

11. If we plotted the first variable with respect to the second variable, what kind of graph would we expect to obtain? Sketch a hypothetical example of such a graph and label the axes.
12. How could we extract the value of the unknown constant quantity from this graph?

13. Which method would yield a more reliable and accurate value for the unknown constant: the "single-point" or "multiple-point" measurement? Explain your reasoning.

14. Take a moment to review the previous eleven questions. You should be able to begin to develop an experimental strategy for accomplishing the stated objective. To solidify this strategy, construct a table which you can use in the laboratory to record the data needed. Be sure to construct your table so that data can easily be transferred from tabular to graphical form.
Static Torque Experiment ---- Lab Help-Sheet

The following is a list of helpful hints and questions that you should look over prior to your lab session and take into the laboratory with you. They will help you to think about and identify good experimental techniques.

REMEMBER: The final objective of the experiment is to determine the unknown magnetic moment of your given dipole.

1. Useful tools:
   - calipers calibrated to at least 1 mm
   - ruler calibrated to at least 1 mm
   - scale calibrated to at least one-tenth of a gram

2. Bring into the laboratory the data tables which you have constructed, but feel free to modify them if you discover a better way of organizing your data.

3. In addition to recording your data in tabular form, it is important to plot your data on graph paper as you perform the experiment. This ensures that if an aberrant data point appears, you will be able to recognize it and repeat the measurement to investigate the validity of that point.

4. Does it matter whether the magnetic field is the dependent or independent variable in the experiment?

5. The mass of the rod and location of its center of mass are constant throughout the experiment. Will these values affect the slope of the straight line data plot? If not, what characteristic of the plot will they affect?

6. Does performing one trial of the experiment or multiple trials of the experiment lead to greater confidence in your result?

7. Does having more data points on the graph lead to greater confidence in your result?

8. REMINDER: The magnetic field is directly proportional to the current as read on the Magnetic Torque ammeter, with the proportionality factor being 0.00137 teslas/amp. To verify this value, approximate the coil magnet as two co-axial current loops, each having a radius of 9.9 cm, and separated by 14.0 cm. Keep in mind that the coils have 195 turns each, and the Magnetic Torque ammeter measures the current passing through one turn. Since coils are in series, the current passing through each turn is identical. However, what if the coils were in parallel? Would the current necessarily be identical in each coil?

SET UP A TABLE TO RECORD YOUR DATA. MAKE GRAPHS USING GRAPH PAPER. SHOW SAMPLE CALCULATIONS. EXPLAIN YOUR RESULT.
**Precessional Motion Experiment** --- Pre-Lab Assignment

**OBJECTIVE:**
A magnetic dipole experiences a net torque in a uniform magnetic field. Under certain conditions, this torque will cause a rotating body to precess. Take advantage of this physical principle to determine the unknown magnetic moment of a given dipole.

**QUESTIONS:**

1. The picture below shows the Magnetic Torque cue ball floating on the air bearing in a uniform magnetic field. This corresponds to the condition when the currents in the top and bottom coils of the Magnetic Torque instrument are flowing in the same direction. Draw on the picture any torques that the ball experiences, and give the magnitude of these torques in algebraic form. (When you’re in the lab, you will observe that in the absence of a magnetic field the cue ball is nearly rotationally balanced. Neither the handle nor the embedded magnet produce a significant gravitational torque.)

![Figure 1](image)

2. What relationship exists between a torque on a body and that body’s change in angular momentum? Express this relationship in algebraic form.

3. Suppose that the ball in Figure 1 was just released from rest. Based on your answer to question #2, describe the motion of the ball.
4. There should be some mention of angular momentum in your answer to question #3. Is the angular momentum vector parallel, anti-parallel, or perpendicular to the torque vector in the situation shown in Figure 2?

5. Does the motion described in question #3 involve changes in the magnitude of the angular momentum? Does the motion involve changes in the direction of the angular momentum?

6. Now suppose that the ball is given a large spin angular momentum about its handle's axis before being released from rest, as shown in the picture below. Is the torque experienced by the ball parallel, anti-parallel, or perpendicular to the spin angular momentum?

Figure 2

7. Will the torque cause a change in the magnitude of the spin angular momentum? Will it cause a change in the direction of the spin angular momentum?

8. Describe the motion of the ball in Figure 2. (Hint: think of how this situation is similar to a gyroscope in the earth’s gravitational field.)
9. Carefully read through the supplemental derivation of the expression for the precessional period of the ball. Look carefully at the final expression. What letters represent physical quantities which we can measure using standard laboratory equipment and/or the Magnetic Torque instrument? (Hint: The magnitude of the magnetic field is directly proportional to the current, with the proportionality factor being 0.00137 teslas/ampere.)

10. What letters represent quantities that can be varied and measured? What letters represent quantities that the experimenter can keep constant, at least during the time it takes to make each separate measurement? Note that you, as the experimenter, may have some options in this experiment as to which quantities to vary and which to keep constant. Make sure to justify your decision.

11. What is the remaining letter that represents an unknown constant quantity?

12. What kind of graph (somehow incorporating the two variables on the two axes) would we like to obtain in order to extract the value of this unknown constant quantity?

13. How would you then extract this unknown constant quantity from the graph?

14. Take a moment to review the previous thirteen questions. After doing so, you should be able to begin to develop an experimental strategy that can be used to achieve the stated objective. Solidify this strategy by constructing a table that can be used in the laboratory to record your needed data. Make sure that the table allows you to easily transfer data from tabular to graphical form.
Precessional Motion Experiment ---- Lab Help-Sheet

The following are some helpful hints and questions that you should read over before the lab session and take into the laboratory with you. They will help you to think about and identify good laboratory techniques.

REMEMBER: The experiment's final objective is to determine the unknown magnetic moment of your given dipole.

1. Bring into the lab the data tables which you have constructed, but feel free to modify them if you encounter a better way of organizing your data.

2. In addition to recording your data in tabular form, it is important to plot your data on graph paper as you perform the experiment. This ensures that if an aberrant data point appears, you will be able to recognize it and repeat the measurement to investigate the validity of that point.

3. Useful tools:
   - calipers
   - scale
   - stopwatch
   - strobe light set to “ON” on Magnetic Torque front panel

4. How do you know when the ball’s spin frequency is equal to the strobe light frequency?

5. Is it easier to vary the precessional period (T) or the magnetic field (B) as the independent variable?

6. Can you devise a way to keep the ball’s spin frequency constant for all measurements throughout the experiment? Is this necessary in your particular experimental strategy?

7. Which yields more confidence in your result and is experimentally easiest to obtain: a low, medium, or high constant spin frequency of the ball?

8. Should there be current passing through the magnet coils continuously throughout the experiment, or should you just turn the current on and off as you need it?

9. Should you time the ball’s precession for one period, or for multiple periods?

10. Should more data be collected for low or high magnetic fields, or equal amounts for both?

11. Does the direction of the field seem to matter for obtaining a better result?

12. Is the angular frequency of the ball’s spin equal to the frequency as read off the strobe display?
13. REMINDER: The magnetic field is related to the current as read on the Magnetic Torque ammeter by the value 0.00137 teslas/amp.

SET UP TABLES TO RECORD DATA. MAKE GRAPHS USING GRAPH PAPER. SHOW SAMPLE CALCULATIONS. EXPLAIN YOUR RESULT.
Supplement to Precessional Motion Experiment — Derivation of Precessional Period

The differential equation for the rotational motion of the ball is:

\[ \tau = \frac{dL}{dt} \]

A side view of the angular momentum vectors is shown below:

If we look from above the spinning ball at the change of the angular momentum for a short time \( \Delta t \), the picture looks like the one below:

Since \( s = r \theta \), we can show that:

\[ \Delta L_x = \Delta \dot{\theta} L_x \sin \theta' \]
\[ \frac{\Delta L_x}{\Delta t} = \frac{\Delta \dot{\theta}}{\Delta t} L_x \sin \theta' \]

as \( \Delta t \to 0 \),

\[ \frac{dL_x}{dt} = \Omega_x L_x \sin \theta' \]

where \( \Omega_x \) is the precessional angular velocity. From our original differential equation,

\[ \frac{dL_x}{dt} = \mu B \sin \theta' \]
So we obtain the expression

\[ \mu B \sin \theta' = \Omega_L \sin \theta' \]

\[ \mu B = \Omega_L L \]

\[ \Omega_p = \frac{\mu}{L} B \]

Since

\[ \Omega_p = \frac{2\pi}{T_p} \]

and

\[ L = I \omega \]

we obtain the expression

\[ T_p = \frac{2\pi \omega}{\mu B} \]